Games of Today

SUDOKU GAME

4				9	1			
		9			7	4	2	5
	5	8	3	4		1	9	
6	9	1						
		3	9	6	4	7		
						9	6	3
	8	7		2	6	5	3	
3	1	5	8			6		
			1	5		8	7	9

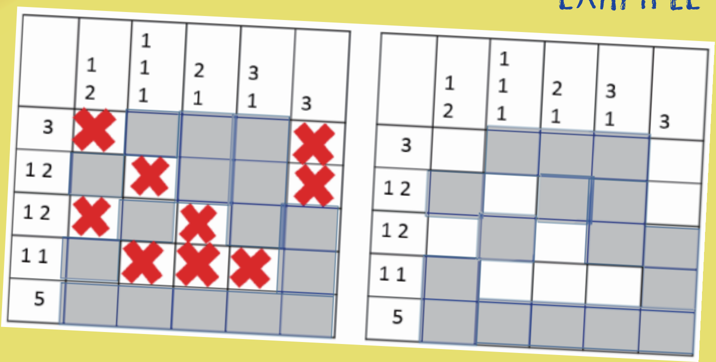
PICROSS

	2	2 3	4 3	2 1 1 1	2 2	1 1	1 2	1 1	2 1	2 1 2 1
1 1 1										
2 2 2										
2 5										
2 1										
1										
1 1 1										
1 2 2										
2 1										
2 3										
2 1										

SUDOKU GAME ANSWER

4	6	2	5	9	1	3	8	7
1	3	9	6	8	7	4	2	5
7	5	8	3	4	2	1	9	6
6	9	1	7	3	8	2	5	4
5	2	3	9	6	4	7	1	8
8	7	4	2	1	5	9	6	3
9	8	7	4	2	6	5	3	1
3	1	5	8	7	9	6	4	2
2	4	6	1	5	3	8	7	9

EXAMPLE



WORD FIND

F	W	B	Q	U	O	L	E	S	N	L	E	Z	W	O
Z	R	T	L	E	I	A	R	B	E	G	L	A	A	Z
H	E	A	F	V	V	D	I	M	M	X	I	M	F	Y
E	Q	O	C	I	D	F	G	T	U	O	X	T	Z	U
N	M	P	V	T	E	Y	H	P	L	P	L	E	B	D
I	S	E	V	A	I	H	T	S	O	I	J	B	E	Q
L	B	R	X	G	L	O	A	Q	V	O	N	N	S	V
R	B	I	G	E	D	X	N	M	O	K	O	C	L	P
E	A	M	Q	N	S	W	G	Q	E	M	X	B	R	F
B	R	E	X	D	W	F	L	R	I	K	F	I	E	S
M	C	T	W	H	O	L	E	N	U	M	B	E	R	C
U	H	E	W	G	E	H	A	T	V	B	D	H	U	Y
N	A	R	G	A	P	T	N	A	C	J	T	B	J	R
K	R	E	W	S	O	E	K	Z	T	L	E	G	N	Q
C	T	F	W	R	S	L	E	L	L	A	R	A	P	X

Find as many mathematical terms as possible in the grid, running in one of eight possible directions horizontally, vertically, or diagonally.

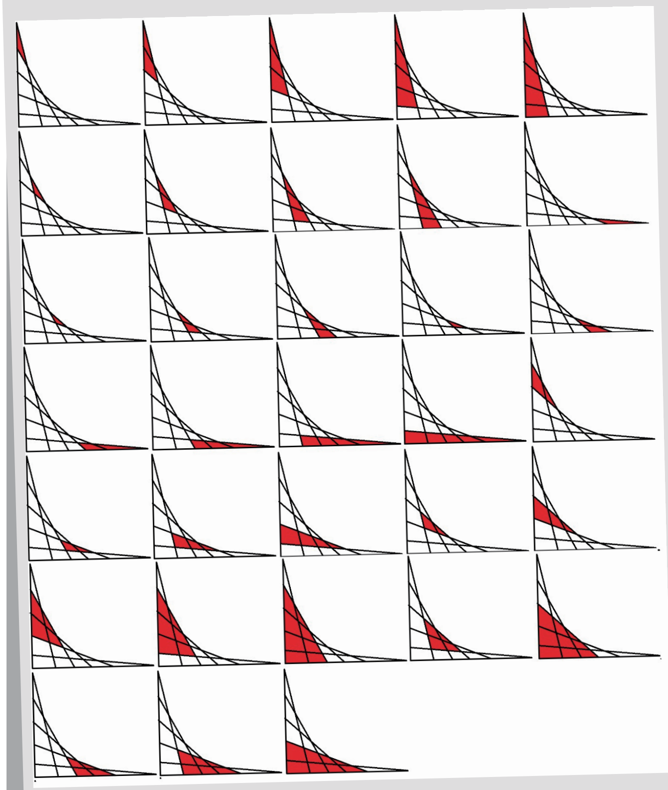
數趣大地 MATHGAZINE

LOGOS ACADEMY March 2019

MATH GARDEN

SEVEN LINES PROBLEM

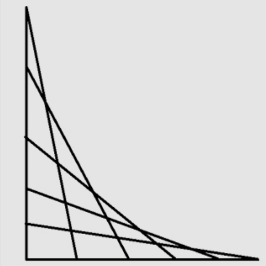
Winner: MS2Y Timothy Choi



PROBLEM

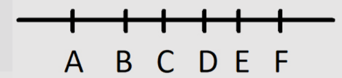
Draw seven non-parallel lines where there are no three lines intersect at a point. (We called the lines are not coincided)

(The set of lines with the above two conditions are called lines in the general position.)



1. In the diagram, each line can cut the other 6 lines by 6 points of intersection and each line segment can be formed by any two points out of 6. i.e. There are 15 line segments in total. The line segments can be counted by: (7-2)+(7-3)+(7-4)+(7-5)+(7-6) = 15

(i.e. AB, BC, CD, DE, EF & AC, BD, CE, DF & AD, BE, CF & AE, BF & AF)



As there are 7 lines, which means there are in total

$$7[(7-2)+(7-3)+(7-4)+(7-5)+(7-6)] = 105$$

line segments.

2. Every three line segments can be used to form triangles uniquely. (as no lines are coincided)

$$\frac{105}{3} = 35$$

Is the number of triangles

3. In general, when there are n lines in the **general position**. The total number of line segments is:

$$n[(n-2)+(n-3)+\ldots+1]$$

where $(n-2)+(n-3)+\cdots+1 = \frac{(n-2)(n-1)}{2}$

i.e. There are $\frac{n(n-2)(n-1)}{2}$ line segments.

Similar as above, every three line segments can be used to form triangles uniquely.

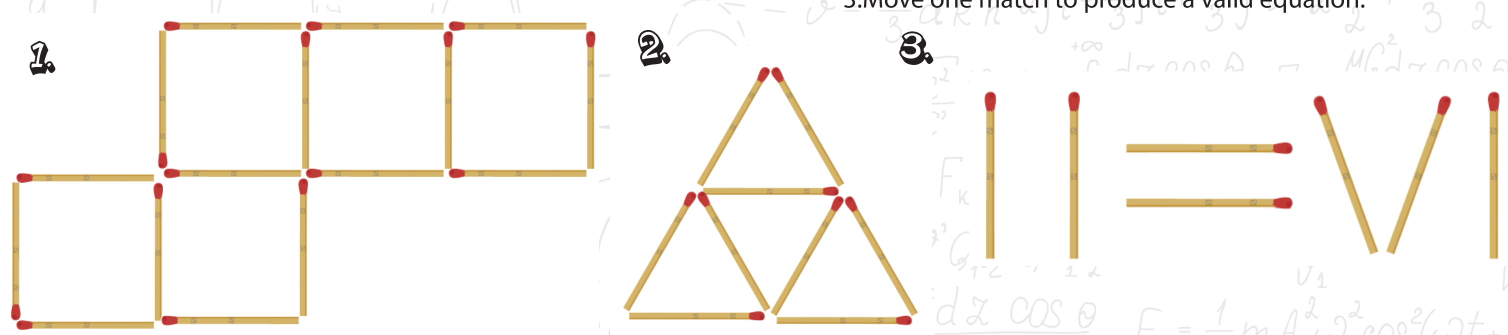
$$\frac{n(n-2)(n-1)}{6}$$

is the number of triangles.

CHALLENGE CORNER

1.Change the positions of 2 matchsticks to reduce the number of squares from 5 to 4.
2.Remove 2 matchsticks and leave 2 equilateral triangles.
3.Move one match to produce a valid equation.

1. 2. 3.



STUDENTS' ZONE

DIVISIBILITY

Lam Arthur MS2T, Chan Cheuk Kin MS2E

A lot of students dislike Mathematics because they memorize a bunch of formulae instead of understanding them. By understanding the proof behind formulae, we can apply these formulae into questions easily.

In this article, we will investigate on the divisibility rule. To begin, let's revise how we can test a number's divisibility.

Divisors	Tests for a number's divisibility
2	The units digit is even
5	The units digit is 5 or 0
3	The sum of all digits is divisible by 3
9	The sum of all digits is divisible by 9
4	The value of the last two digits is divisible by 4
8	The number can be divided by 8 if it satisfies one of the followings: (I) The hundreds digit is even and the value of the last two digits is divisible by 8 (e.g. 208, 416) **OR** (II) The hundreds digit is odd and the value of the last two digits is divisible by 4 but not by 8 (e.g. 120, 728)

Now consider the test for the divisor 3 of any number, e.g. 1314, since 1+3+1+4=9 and 9 is divisible by 3, therefore 1314 is divisible by 3.

But do you understand how this test works? Imagine you have a number 'abcd'. (where a is the thousands digit, b is the hundreds digit, c is the tens digit and d is the units digit. To avoid confusion with $a\times b\times c\times d$, we usually use $\overline{abcd}$) By using algebra, it can be written as:

$$\overline{abcd}=1000a+100b+10c+d$$

From $1000a+100b+10c+d$, $1000a$ can be written as $999a+a$, $100b$ can be written as $99b+b$, and $10c$ can be written as $9c+c$.

$$\begin{aligned}\overline{abcd}&=1000a+100b+10c+d\\&=999a+a+99b+b+9c+c+d\\&=(999a+99b+9c)+(a+b+c+d)\\&=9(111a+11b+c)+(a+b+c+d)\end{aligned}$$

This is the proof of the test for the divisor 3 of any number, can you find the proof of the test for the divisors 4, 9 and 8 using the proof above?

If that happens to be too easy for you, then, can you find the proof of the test for the divisor 7 ("cut-tail method") using the proof above?

For example, 3969 is divisible by 7.

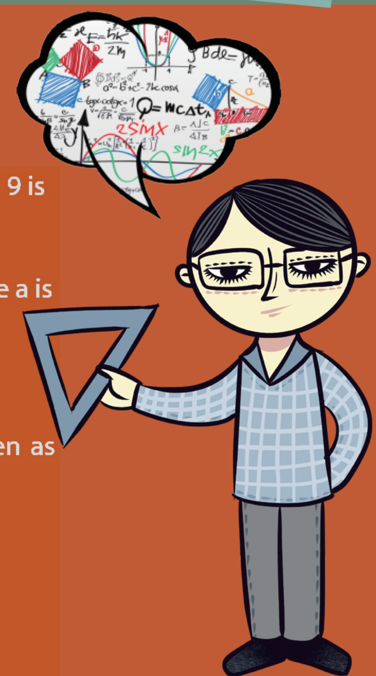
Test method: "Cut" away the units digit '9', then subtract 9x2 from the remaining value 396,

$$\begin{array}{r}396\\-18\\\hline 378\end{array}$$

Repeat the process above, "cut" away the units digit '8', then subtract 8x2 from the remaining value 37,

$$\begin{array}{r}37\\-16\\\hline 21\end{array}$$

Since 21 is divisible by 7, therefore 3969 is divisible by 7.



CHALLENGE CORNER

MAKING A FOOL OF MR. CHOW THREE TIMES

One day, homeroom teacher Mr. Chow asked Ming, the monitor of the class, to collect the students' maths assignment books and hand them in. The next day, he asked Ming, 'Ming, did everyone hand in their homework? How many assignment books have you collected?'

Ming realized that he would be scolded if Mr Chow knew he failed to collect the assignments from everyone. So he came up with an ingenious answer. 'Well, if we group the assignment books into groups of 3, there will be 2 left. If we group them into groups of 5, there will be 3 left. If we group them into groups of 7, there will be 2 left. So, how many assignment books do you think I have collected?'

TEACHER'S SHARING

EXTENSIVE APPLICATION OF MODUS PONENS IN PLANE GEOMETRY

Dr. Percy Kwok

In formal logic, the strict definition of modus ponens (MP) is as follows:

p (given condition) implies q (conclusion) and p is asserted to be true, therefore q must be true. $(p\rightarrow q)$

In fact, modus ponens is often used in our daily life. Here are two examples:

1. $p\rightarrow q$, where

   Given condition p: a student is caught using his mobile phone in class

   Conclusion q: the student receives a warning ticket from the School Discipline Office

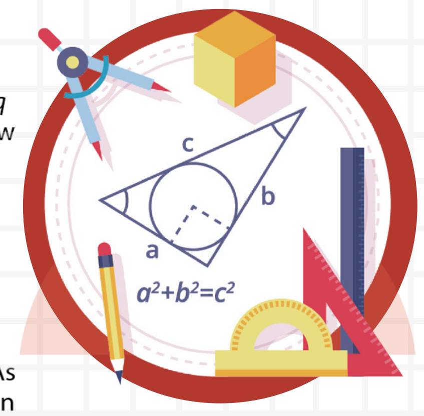
   Now you are caught using your mobile phone in class. Because condition p leads to conclusion q and condition p is met, by applying modus ponens, q must be true as the result of p. So you know immediately that the School Discipline Office will issue a warning ticket to you.

2. $p\rightarrow q$, where

   Given condition p: It rains today

   Conclusion q: I bring an umbrella

   Now it is raining today. By applying modus ponens, I will bring an umbrella.



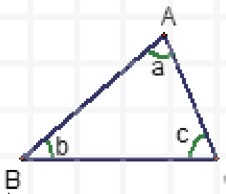
In plane geometry, we apply modus ponens unconsciously when we try to use certain theorem. As long as we can confirm the given condition of a theorem is met, we can safely derive a conclusion based on the theorem. Let's have a look at the following example.

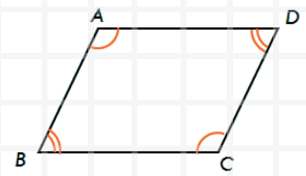
** Theorem of Angle Sum of a Triangle: The sum of the three interior angles of any triangle equals to 180˚.

In this theorem, the given condition p is the statement that there are three measurable, unrepeated interior angles in any triangle, while the conclusion q is the statement that the sum of the three interior angles must equal to 180˚.

If we can identify a triangle and its three interior angles, which means p is true, we can immediately apply MP and get an equality that shows the sum of three angles is 180˚, which is the q of this theorem.

When we use this theorem, we must ensure the triangle is on a plane, and none of the three interior angles is repeated.





Do $p\rightarrow q$ and $q\rightarrow p$ have the same meaning?

The answer is no. In fact, "if p then q" and "if q then p" are logically distinct, which can be clearly demonstrated using one of the previous examples.

Let's assume

Statement p: It rains today

Statement q: I bring an umbrella

The meaning of $p\rightarrow q$ is to take certain action (bringing an umbrella in this case) based on the weather condition (raining). However, the meaning of $q\rightarrow p$ is to guess the weather condition (raining in this case) that leads to certain action (bringing an umbrella). Logically, they are completely different.

In plane geometry, you can find many situations where $p\rightarrow q$ and $q\rightarrow p$ have very different meanings. Let's look at the following two theorems.

** Theorem of Opposite Angles of Parallelogram: In any parallelogram (p), the opposite angles are congruent (q).

To use this theorem in solving a geometry problem, we have to first identify a parallelogram (p). By applying MP, we can then derive that the opposite angles in it must be congruent (q). Please note the assumption here is that the concerned quadrilateral is on a plane.

** Theorem of Opposite Angles Equal: In any quadrilateral, if both pairs of opposite angles are congruent (p), the quadrilateral must be a parallelogram (q).

To use this theorem in solving a geometry problem, we have to first identify a quadrilateral and know, either as given or by proof, that its two pairs of opposite angles are congruent (p). By applying MP, we can conclude that the quadrilateral is a parallelogram (q). Please note the assumption here is that the concerned quadrilateral is on a plane.

The two theorems above seem so similar that lots of students mix them up. However, the following MP structure can help us easily discriminate between the fundamentally opposite geometry meanings of the two theorems.

** Theorem of Opposite Angles of Parallelogram:

Given condition p: a parallelogram

Conclusion q: two sets of opposite angles in it are congruent

** Theorem of Opposite Angles Equal:

Given Condition p: both pairs of opposite angles in a quadrilateral are congruent

Conclusion q: the quadrilateral must be a parallelogram

Now that we know $p\rightarrow q$ and $q\rightarrow p$ have different meanings, can we conclude $q\rightarrow p$ is true if we know $p\rightarrow q$ is true? You can find the answer by trying to solve the following problem.

Assume ABCD is a quadrilateral on a plane.

Statement p: ABCD is a parallelogram

Statement q: AB=CD and BC//DA

Determine whether

- $p\rightarrow q$ is true
- $q\rightarrow p$ is true

When we solve geometry problems, it is important to distinguish the conclusion of a theorem from the given condition of the theorem.